Hoofdstuk 15: Toepassingen

15.1 Oppervlakten en afstanden bij grafieken

Opgave 1:

a. $x_Q = 1\frac{1}{2}$

$y_Q = \sqrt{2}$

$Opp = 1\frac{1}{2}\sqrt{2}$

b. $x_Q = p$

$y_Q = \sqrt{5-2p}$

$Opp = OP \cdot PQ = p\sqrt{5-2p}$

c. $y_1 = x\sqrt{5-2x}$

de optie maximum geeft $y = 2{,}15$, dus de maximale oppervlakte is 2,15

Opgave 2:

a. $x_P = p$

$y_P = \sqrt{3-p}$

$Opp(\Delta OPQ) = \frac{1}{2} \cdot OQ \cdot PQ = \frac{1}{2}p\sqrt{3-p}$

d. $\frac{dA}{dp} = \frac{1}{2} \cdot \sqrt{3-p} + \frac{1}{2}p \cdot \frac{1}{2\sqrt{3-p}} \cdot -1$

$= \frac{1}{2}\sqrt{3-p} - \frac{p}{4\sqrt{3-p}}$

$= \frac{2(3-p)}{4\sqrt{3-p}} - \frac{p}{4\sqrt{3-p}}$

$= \frac{6-2p-p}{4\sqrt{3-p}}$

$= \frac{6-3p}{4\sqrt{3-p}}$

c. $A' = \frac{6-3p}{4\sqrt{3-p}} = 0$

$6-3p = 0$

$-3p = -6$

$p = 2$

$A = 1$

d. $OP = \sqrt{OQ^2 + PQ^2} = \sqrt{p^2 + (\sqrt{3-p})^2} = \sqrt{p^2 + 3 - p} = \sqrt{p^2 - p + 3}$

e. $L' = \frac{1}{2\sqrt{p^2 - p + 3}} \cdot (2p-1) = \frac{2p-1}{2\sqrt{p^2 - p + 3}} = 0$

$2p - 1 = 0$

$2p = 1$

$p = \frac{1}{2}$

$L = \sqrt{2\tfrac{3}{4}} = \tfrac{1}{2}\sqrt{11}$

Opgave 3:

a. $Opp(\Delta OSP) = \tfrac{1}{2} \cdot 4 \cdot p\sqrt{8-2p} = 2p\sqrt{8-2p}$

$$Opp' = 2 \cdot \sqrt{8-2p} + 2p \cdot \frac{1}{2\sqrt{8-2p}} \cdot -2$$

$$= 2\sqrt{8-2p} - \frac{2p}{\sqrt{8-2p}} = 0$$

$$2\sqrt{8-2p} = \frac{2p}{\sqrt{8-2p}}$$

$2(8-2p) = 2p$

$16 - 4p = 2p$

$-6p = -16$

$p = \tfrac{8}{3}$

$Opp = 5\tfrac{1}{3}\sqrt{2\tfrac{2}{3}} = 5\tfrac{1}{3} \cdot \tfrac{2}{3}\sqrt{6} = \tfrac{32}{9}\sqrt{6}$

b. $Opp(\Delta QSP) = \tfrac{1}{2} \cdot QS \cdot QP = \tfrac{1}{2}(4-p) \cdot p\sqrt{8-2p} = (2p - \tfrac{1}{2}p^2)\sqrt{8-2p}$

c. $$\frac{dA}{dp} = (2-p)\sqrt{8-2p} + (2p - \tfrac{1}{2}p^2) \cdot \frac{1}{2\sqrt{8-2p}} \cdot -2$$

$$= (2-p)\sqrt{8-2p} - \frac{2p - \tfrac{1}{2}p^2}{\sqrt{8-2p}}$$

$$= \frac{(2-p)(8-2p)}{\sqrt{8-2p}} - \frac{2p - \tfrac{1}{2}p^2}{\sqrt{8-2p}}$$

$$= \frac{16 - 12p + 2p^2 - 2p + \tfrac{1}{2}p^2}{\sqrt{8-2p}}$$

$$= \frac{2\tfrac{1}{2}p^2 - 14p + 16}{\sqrt{8-2p}}$$

$$= \frac{5p^2 - 28p + 32}{2\sqrt{8-2p}}$$

d. $$A' = \frac{5p^2 - 28p + 32}{2\sqrt{8-2p}} = 0$$

$5p^2 - 28p + 32 = 0$

$$p = \frac{28 \pm \sqrt{144}}{10} = \frac{28 \pm 12}{10}$$

$p = 1{,}6 \quad \vee \quad p = 4$ (vervalt)

$Opp = 4{,}21$

Opgave 4:

a. $Opp(\Delta OPQ) = \tfrac{1}{2} \cdot 2p \cdot (3 - \tfrac{1}{2}p^2) = 3p - \tfrac{1}{2}p^3$

b. $Opp' = 3 - 1\frac{1}{2}p^2 = 0$

$-1\frac{1}{2}p^2 = -3$

$p^2 = 2$

$p = \sqrt{2} \quad \vee \quad p = -\sqrt{2}$ (vervalt)

$Opp(\Delta OPQ) = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

c. $OP = \sqrt{p^2 + (3 - \frac{1}{2}p^2)^2} = \sqrt{p^2 + 9 - 3p^2 + \frac{1}{4}p^4} = \sqrt{\frac{1}{4}p^4 - 2p^2 + 9}$

$OP' = \dfrac{1}{2\sqrt{\frac{1}{4}p^4 - 2p^2 + 9}} \cdot (p^3 - 4p) = 0$

$p^3 - 4p = 0$

$p(p^2 - 4) = 0$

$p = 0 \quad \vee p^2 = 4$

$p = 0 \quad \vee \quad p = 2 \quad \vee \quad p = -2$

dus $p = 2$

$OP = \sqrt{5}$

Opgave 5:

a. $AP = \sqrt{(4-p)^2 + (p\sqrt{p})^2} = \sqrt{16 - 8p + p^2 + p^3}$

b. $AP' = \dfrac{1}{2\sqrt{p^3 + p^2 - 8p + 16}} \cdot (3p^2 + 2p - 8) = 0$

$3p^2 + 2p - 8 = 0$

$p = \dfrac{-2 \pm \sqrt{100}}{6} = \dfrac{-2 \pm 10}{6}$

$p = \frac{4}{3} \quad \vee \quad p = -2$ (vervalt)

$P = (\frac{4}{3}, \frac{4}{3}\sqrt{\frac{4}{3}}) = (\frac{4}{3}, \frac{8}{9}\sqrt{3})$

Opgave 6:

a. $Opp(\Delta OSP) = \frac{1}{2} \cdot 8 \cdot p \cdot \sqrt[3]{8-p} = 4p \cdot \sqrt[3]{8-p}$

$Opp' = 4 \cdot \sqrt[3]{8-p} + 4p \cdot \frac{1}{3} \cdot (8-p)^{-\frac{2}{3}} \cdot -1$

$\quad = 4 \cdot \sqrt[3]{8-p} - \dfrac{4p}{3 \cdot \sqrt[3]{(8-p)^2}} = 0$

$4 \cdot \sqrt[3]{8-p} = \dfrac{4p}{3 \cdot \sqrt[3]{(8-p)^2}}$

$12(8-p) = 4p$

$96 - 12p = 4p$

$-16p = -96$

$p = 6$

$Opp = 24 \cdot \sqrt[3]{2}$

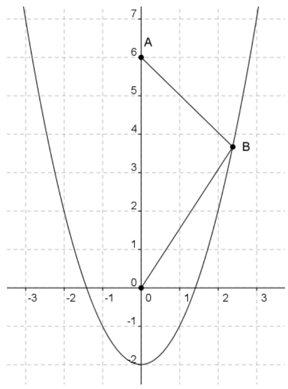
b. $Opp(\Delta OPQ) = \frac{1}{2} \cdot p \cdot p \cdot \sqrt[3]{8-p} = \frac{1}{2}p^2 \cdot \sqrt[3]{8-p}$

c. $A' = p \cdot \sqrt[3]{8-p} + \frac{1}{2}p^2 \cdot \frac{1}{3}(8-p)^{-\frac{2}{3}} \cdot -1$

$= p \cdot \sqrt[3]{8-p} - \dfrac{p^2}{6 \cdot \sqrt[3]{(8-p)^2}}$

$A'(7) = 7 \cdot \sqrt[3]{1} - \dfrac{7^2}{6 \cdot \sqrt[3]{1}} = 7 - \frac{49}{6} = -1\frac{1}{6} \neq 0$ dus nee

Opgave 7:



$B = (p, p^2 - 2)$

$OB = \sqrt{p^2 + (p^2-2)^2}$

$= \sqrt{p^2 + p^4 - 4p^2 + 4}$

$= \sqrt{p^4 - 3p^2 + 4}$

$AB = \sqrt{p^2 + (6 - (p^2-2))^2}$

$= \sqrt{p^2 + (8 - p^2)^2}$

$= \sqrt{p^2 + 64 - 16p^2 + p^4}$

$= \sqrt{p^4 - 15p^2 + 64}$

$L = \sqrt{p^4 - 3p^2 + 4} + \sqrt{p^4 - 15p^2 + 64}$

neem $y_1 = \sqrt{p^4 - 3p^2 + 4} + \sqrt{p^4 - 15p^2 + 64}$ de optie minimum geeft $x = 1{,}85$ en $y = 7{,}27$
dus $L = 7{,}27$

Opgave 8:

a. $y_A = f(-3) = 3$

$y_B = g(-3) = -1\frac{1}{2}$

$L = 3 - -1\frac{1}{2} = 4\frac{1}{2}$

b. $y_A = f(p) = \sqrt{2p+15}$

$y_B = g(p) = \frac{1}{2}p$

$L = y_A - y_B = \sqrt{2p+15} - \frac{1}{2}p$

c. $y_1 = \sqrt{2x+15} - \frac{1}{2}x$ de optie maximum geeft $x = -5{,}5$ dus $p = -5{,}5$

Opgave 9:

a. $y_A = f(p) = \sqrt{6p+12}$

$y_B = g(p) = p + 2$

$L = y_A - y_B = \sqrt{6p+12} - (p+2) = \sqrt{6p+12} - p - 2$

b. $L' = \dfrac{1}{2\sqrt{6p+12}} \cdot 6 - 1 = \dfrac{3}{\sqrt{6p+12}} - 1 = 0$

$\dfrac{3}{\sqrt{6p+12}} = 1$

$\sqrt{6p+12} = 3$

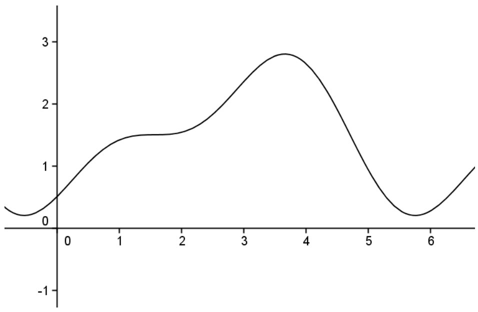
$6p + 12 = 9$
$6p = -3$
$p = -\frac{1}{2}$
$L = 1\frac{1}{2}$

Opgave 10:

$y_C = f(p) = \sqrt{p^2 + 4}$
$y_D = g(p) = \frac{1}{2}p + 5$
$CD = y_D - y_C = \frac{1}{2}p + 5 - \sqrt{p^2 + 4}$
$CD' = \frac{1}{2} - \frac{1}{2\sqrt{p^2+4}} \cdot 2p = \frac{1}{2} - \frac{p}{\sqrt{p^2+4}} = 0$
neem $y_1 = \frac{1}{2} - \frac{x}{\sqrt{x^2+4}}$ de optie zero geeft: $x = 1{,}15$
$CD = 3{,}27$

Opgave 11:

$ab = y_A - y_B = \frac{1}{2}\sin 2x - (\cos x - 1\frac{1}{2}) = \frac{1}{2}\sin 2x - \cos x + 1\frac{1}{2}$
$AB' = \frac{1}{2} \cdot \cos 2x \cdot 2 + \sin x = \cos 2x + \sin x = 0$
$1 - 2\sin^2 x + \sin x = 0$
$\sin^2 x - \frac{1}{2}\sin x - \frac{1}{2} = 0$
$(\sin x - 1)(\sin x + \frac{1}{2}) = 0$
$\sin x = 1 \quad \vee \quad \sin x = -\frac{1}{2}$
dus $x = 1\frac{1}{6}\pi$
$AB = 1\frac{1}{2} + \frac{3}{4}\sqrt{3}$



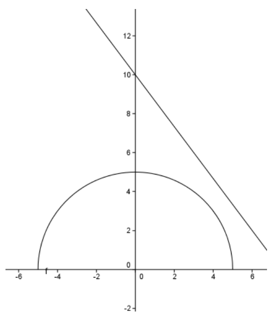
Opgave 12:

a. $AB = g(x) - f(x) = -\frac{4}{3}x + 10 - \sqrt{25 - x^2}$

$AB' = -\frac{4}{3} - \frac{1}{2\sqrt{25-x^2}} \cdot -2x = -\frac{4}{3} + \frac{x}{\sqrt{25-x^2}} = 0$

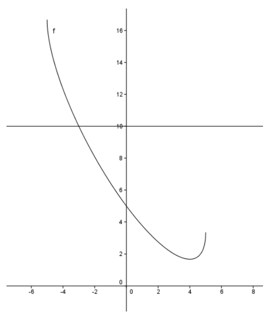
neem $y_1 = -\frac{4}{3} + \frac{x}{\sqrt{25-x^2}}$ de optie zero geeft $x = 4$

$AB = 1\frac{2}{3}$



b. $-\frac{4}{3}x + 10 - \sqrt{25 - x^2} > 10$

$-\sqrt{25 - x^2} = \frac{4}{3}x$
$25 - x^2 = \frac{16}{9}x^2$
$25 = \frac{25}{9}x^2$
$x^2 = 9$
$x = -3 \quad \vee \quad x = 3$ (vervalt)
$D_f = [-5{,}5]$ dus $-5 \le x < -3$



Opgave 13:

$L = f(x) - g(x) = \ln(2x+5) - \tfrac{1}{2}x$

$L' = \dfrac{1}{2x+5} \cdot 2 - \tfrac{1}{2} = \dfrac{2}{2x+5} - \tfrac{1}{2} = 0$

$\dfrac{2}{2x+5} = \tfrac{1}{2}$

$2x + 5 = 4$

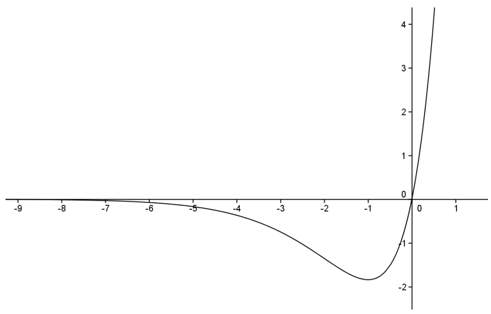
$2x = -1$

$x = -\tfrac{1}{2}$

$L = \ln 4 + \tfrac{1}{4}$

Opgave 14:

a. $f(x) = 5x \cdot e^x$



$f'(x) = 5 \cdot e^x + 5x \cdot e^x = (5+5x) \cdot e^x = 0$

$5 + 5x = 0 \quad \vee \quad e^x = 0$

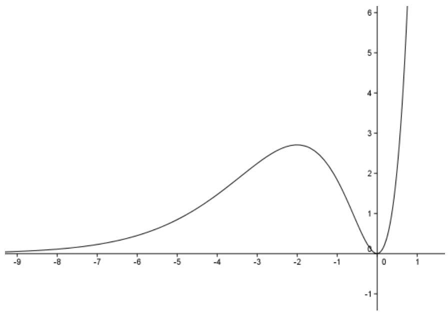
$5x = -5$

$x = -1$

$y = -\tfrac{5}{e}$

$B_f = [-\tfrac{5}{e}, \rightarrow\rangle$

b. $g(x) = 5x^2 \cdot e^x$



$g'(x) = 10x \cdot e^x + 5x^2 \cdot e^x = (10x + 5x^2) \cdot e^x = 0$

$5x^2 + 10x = 0 \quad \vee \quad e^x = 0$

$5x(x+2) = 0$

$x = 0 \quad \vee \quad x = -2$

max $g(-2) = \tfrac{20}{e^2}$

min $g(0) = 0$

c. $AB = g(x) - f(x) = 5x^2 \cdot e^x - 5x \cdot e^x = (5x^2 - 5x) \cdot e^x$

$AB' = (10x - 5) \cdot e^x + (5x^2 - 5x) \cdot e^x = (5x^2 + 5x - 5) \cdot e^x = 0$

$5x^2 + 5x - 5 = 0 \quad \vee \quad e^x = 0$

$x^2 + x - 1 = 0$

$x = \dfrac{-1 \pm \sqrt{5}}{2}$

$p = -\tfrac{1}{2} - \tfrac{1}{2}\sqrt{5}$

d. $CD = f(x) - g(x) = 5x \cdot e^x - 5x^2 \cdot e^x$

neem $y_1 = 5x \cdot e^x - 5x^2 \cdot e^x$ de optie maximum geeft $x = 0{,}618$ en $y = 2{,}190$

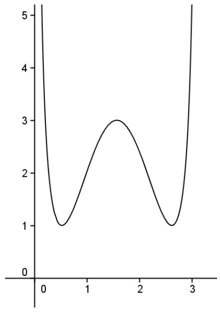
dus $CD = 2{,}190$

Opgave 15:

a. $AB = f(x) - g_4(x) = \dfrac{1}{\sin x} - (4\cos^2 x - 2) = \dfrac{1}{\sin x} - 4\cos^2 x + 2 = (\sin x)^{-1} - 4\cos^2 x + 2$

$AB' = -(\sin x)^{-2} \cdot \cos x - 8\cos x \cdot -\sin x = \dfrac{-\cos x}{\sin^2 x} + 8\sin x \cdot \cos x = 0$

$$8\sin x\cos x=\frac{\cos x}{\sin^2 x}$$
$$8\sin^3 x\cos x=\cos x$$
$$\cos x=0 \quad \vee \quad 8\sin^3 x=1$$
$$\cos x=0 \quad \sin^3 x=\tfrac{1}{8}$$
$$\cos x=0 \quad \vee \quad \sin x=\tfrac{1}{2}$$
$$x=\tfrac{1}{2}\pi \quad \vee \quad x=\tfrac{1}{6}\pi \vee \quad x=\tfrac{5}{6}\pi$$
$$q=\tfrac{1}{6}\pi \quad \vee \quad q=\tfrac{5}{6}\pi$$



b.
$$\begin{cases} f(x)=g(x) \\ f'(x)=g'(x) \end{cases}$$
$$\begin{cases} \dfrac{1}{\sin x}=p\cdot\cos^2 x-2 \\ \dfrac{-\cos x}{\sin^2 x}=2p\cdot\cos x\cdot -\sin x \end{cases}$$
$$\frac{-\cos x}{\sin^2 x}=-2p\cos x\sin x$$
$$\cos x=0 \quad \vee \quad \frac{1}{\sin^2 x}=2p\sin x$$
$$p=\frac{1}{2\sin^3 x}$$
$$\frac{1}{\sin x}=\frac{1}{2\sin^3 x}\cdot\cos^2 x-2$$
neem $y_1=\dfrac{1}{\sin x}$ en $y_2=\dfrac{\cos^2 x}{2\sin^3 x}-2$ intersect geeft $x=0{,}4728 \quad \vee \quad x=2{,}6688$

$p=5{,}29$

15.2 Optimaliseringsproblemen

Opgave 16:

$Inh = 2x \cdot x \cdot h = 2x^2 h = 40$

$h = \frac{40}{2x^2} = \frac{20}{x^2}$

Opgave 17:

a. $Inh = 2x^2 h = 72$

$h = \frac{36}{x^2}$

$K = 2x^2 \cdot 0{,}4 + 2 \cdot 2x \cdot h \cdot 0{,}2 + 2 \cdot x \cdot h \cdot 0{,}2$

$= 0{,}8x^2 + 0{,}8xh + 0{,}4xh$

$= 0{,}8x^2 + 1{,}2xh$

$= 0{,}8x^2 + 1{,}2x \cdot \frac{36}{x^2}$

$= 0{,}8x^2 + \frac{43{,}2}{x}$

b. $K' = 1{,}6x - \frac{43{,}2}{x^2} = 0$

$1{,}6x = \frac{43{,}2}{x^2}$

$1{,}6x^3 = 43{,}2$

$x^3 = 27$

$x = 3$

$h = 4$

$K = 21{,}6$ euro

Opgave 18:

a. $Inh = x^2 h = 16$

$h = \frac{16}{x^2}$

$M = x^2 + 4xh = x^2 + 4x \cdot \frac{16}{x^2} = x^2 + \frac{64}{x}$

b. $M' = 2x - \frac{64}{x^2} = 0$

$2x = \frac{64}{x^2}$

$2x^3 = 64$

$x^3 = 32$

$x = \sqrt[3]{32} = 3{,}17$ dm

$h = 1{,}59$ dm

Opgave 19:

a. $Inh = \pi r^2 h = 1000$

$h = \frac{1000}{\pi r^2}$

$Opp = 2 \cdot \pi r^2 + 2\pi r \cdot h = 2\pi r^2 + 2\pi r \cdot \frac{1000}{\pi r^2} = 2\pi r^2 + \frac{2000}{r}$

b. $Opp' = 4\pi r - \frac{2000}{r^2} = 0$

$4\pi r = \frac{2000}{r^2}$

$4\pi r^3 = 2000$

$r^3 = \frac{500}{\pi}$

$r = \sqrt[3]{\frac{500}{\pi}} = 5{,}4 \text{ cm}$

$h = 10{,}8 \text{ cm}$

Opgave 20:

a. $Opp = xy = 1200$

$y = \frac{1200}{x}$

$K = 15x + 15y + 60y = 15x + 75y = 15x + 75 \cdot \frac{1200}{x} = 15x + \frac{90000}{x}$

b. $K' = 15 - \frac{90000}{x^2} = 0$

$15 = \frac{90000}{x^2}$

$15x^2 = 90000$

$x^2 = 6000$

$x = 77{,}5 \text{ m}$

$y = 15{,}5 \text{ m}$

$K = 2323{,}79$

c. $K = 2500$ dus $15x + \frac{90000}{x} = 2500$

$y_1 = 15x + \frac{90000}{x}$ en $y_2 = 2500$ intersect geeft: $x = 52{,}5 \quad \vee \quad x = 114{,}1$

dus $x = 52{,}6$ en $y = 22{,}8$

Opgave 21:

a. $Inh = \pi r^2 h = 500$

$h = \frac{500}{\pi r^2}$

$Opp_{glas} = \pi r^2 + 2\pi r \cdot h = \pi r^2 + 2\pi r \cdot \frac{500}{\pi r^2} = \pi r^2 + \frac{1000}{r}$

$Opp_{deksel} = \pi r^2 + 2\pi r \cdot 1 = 2\pi r^2 + 2\pi r$

$K = \left(\pi r^2 + \frac{1000}{r}\right) \cdot a + (\pi r^2 + 2\pi r) \cdot 2a$

$= \pi a \cdot r^2 + \frac{1000a}{r} + 2\pi a \cdot r^2 + 4\pi a \cdot r$

$= 3\pi a r^2 + 4\pi a r + \frac{1000a}{r}$

b. $K' = 6\pi a r + 4\pi a - \frac{1000a}{r^2} = 0$

$6\pi r + 4\pi - \frac{1000}{r^2} = 0$

$y_1 = 6\pi x + 4\pi - \frac{1000}{x^2}$ optie zero geeft: $x = 3{,}5$

dus $r = 3{,}5$ en $h = 12{,}6$

Opgave 22:

a. $v_{boot} = v - 3$

$t = \frac{s}{v_{boot}} = \frac{5}{v-3}$

b. $K_{uur} = c \cdot v^2$

$K = K_{uur} \cdot t = c v^2 \cdot \frac{5}{v-3} = \frac{5 c v^2}{v-3}$

c. $K' = \frac{(v-3) \cdot 10 c v - 5 c v^2 \cdot 1}{(v-3)^2}$

$= \frac{10 c v^2 - 30 c v - 5 c v^2}{(v-3)^2}$

$= \frac{5 c v^2 - 30 c v}{(v-3)^2} = 0$

$5 c v^2 - 30 c v = 0$

$5 c v (v - 6) = 0$

$5 c v = 0 \quad \vee \quad v = 6$

k.n. dus $v = 6 \, {}^{km}\!/\!_{uur}$

Opgave 23:

a. $2AC + AB = 12$

$2AC + x = 12$

$2AC = 12 - x$

$AC = 6 - \frac{1}{2}x$

b. $CD = \sqrt{AC^2 - AD^2}$

$= \sqrt{(6 - \frac{1}{2}x)^2 - (\frac{1}{2}x)^2}$

$= \sqrt{36 - 6x + \frac{1}{4}x^2 - \frac{1}{4}x^2} = \sqrt{36 - 6x}$

c. $Opp = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} x\sqrt{36-6x}$

d. $\frac{dO}{dx} = \frac{1}{2}\sqrt{36-6x} + \frac{1}{2}x \cdot \frac{1}{2\sqrt{36-6x}} \cdot -6$

$= \frac{1}{2}\sqrt{36-6x} - \frac{3x}{2\sqrt{36-6x}}$

$= \frac{36-6x}{2\sqrt{36-6x}} - \frac{3x}{2\sqrt{36-6x}}$

$= \frac{36-9x}{2\sqrt{36-6x}}$

e. $\frac{36-9x}{2\sqrt{36-6x}} = 0$

$36-9x = 0$

$-9x = -36$

$x = 4$

$Opp = 2\sqrt{12} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$

Opgave 24:

a. $K = (200-x) \cdot 12 + 2 \cdot 10 \cdot \sqrt{x^2 + 3600}$

$= 2400 - 12x + 20\sqrt{x^2+3600}$

b. $K' = -12 + 20 \cdot \frac{1}{2\sqrt{x^2+3600}} \cdot 2x$

$= -12 + \frac{20x}{\sqrt{x^2+3600}} = 0$

$\frac{20x}{\sqrt{x^2+3600}} = 12$

$20x = 12\sqrt{x^2+3600}$

$400x^2 = 144(x^2+3600)$

$400x^2 = 144x^2 + 518400$

$256x^2 = 518400$

$x^2 = 2025$

$x = 45$

$K = 3360$ euro

Opgave 25:

a. $AB' = \sqrt{500^2 + 200^2} = \sqrt{290000}$

$K = 100 \cdot \sqrt{290000} + 150 \cdot 100 = 68852$ euro

b. $AB = \sqrt{500^2 + 300^2} = \sqrt{340000}$

$AC = \frac{2}{3}\sqrt{340000}$ en $BC = \frac{1}{3}\sqrt{340000}$

$K = 100 \cdot 23\sqrt{340000} + 150 \cdot \frac{1}{3}\sqrt{340000} = 68028$ euro

c. $AP = \sqrt{x^2 + 200^2} = \sqrt{x^2 + 40000}$

$BP = \sqrt{(500-x)^2 + 100^2} = \sqrt{x^2 - 1000x + 260000}$

$K = 100 \cdot \sqrt{x^2 + 40000} + 150 \cdot \sqrt{x^2 - 1000x + 260000}$

$y_1 = 100 \cdot \sqrt{x^2 + 40000} + 150 \cdot \sqrt{x^2 - 1000x + 260000}$

optie minimum geeft: $x = 424{,}4$ en $y = 65721$

dus de minimale kosten zijn 65721 euro

Opgave 26:

a. $AA' = 0{,}1\,\text{km}$

$AP = \sqrt{x^2 + 0{,}1^2} = \sqrt{x^2 + 0{,}01}$

$BP = \sqrt{(0{,}4-x)^2 + 0{,}2^2} = \sqrt{x^2 - 0{,}8x + 0{,}16 + 0{,}04} = \sqrt{x^2 - 0{,}8x + 0{,}2}$

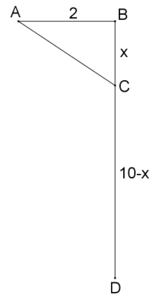
$t = \dfrac{AP}{18} + \dfrac{BP}{12} = \frac{1}{18}\sqrt{x^2 + 0{,}01} + \frac{1}{12}\sqrt{x^2 - 0{,}8x + 0{,}2}$

b. $y_1 = \dfrac{AP}{18} + \dfrac{BP}{12} = \frac{1}{18}\sqrt{x^2 + 0{,}01} + \frac{1}{12}\sqrt{x^2 - 0{,}8x + 0{,}2}$

optie minimum geeft: $x = 0{,}243$ en $y = 0{,}0358$

dus $t = 0{,}0358\,\text{uur} = 128{,}8\,\text{sec}$

Opgave 27:



a. $AC = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$

$CD = 10 - x$

$t = \dfrac{AC}{4} + \dfrac{CD}{12} = \frac{1}{4}\sqrt{x^2 + 4} + \frac{1}{12}(10 - x) = \frac{1}{4}\sqrt{x^2 + 4} + \frac{5}{6} - \frac{1}{12}x$

b. $t' = \frac{1}{4} \cdot \dfrac{1}{2\sqrt{x^2 + 4}} \cdot 2x - \frac{1}{12} = \dfrac{x}{4\sqrt{x^2 + 4}} - \frac{1}{12} = 0$

$\dfrac{x}{4\sqrt{x^2 + 4}} = \frac{1}{12}$

$12x = 4\sqrt{x^2 + 4}$

$3x = \sqrt{x^2 + 4}$

$9x^2 = x^2 + 4$

$8x^2 = 4$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}$

Opgave 28:

a. $DP = 20 - x$

$AD = \sqrt{DP^2 - AP^2} = \sqrt{x^2 - 40x + 400 - x^2} = \sqrt{400 - 40x}$

$Opp(\Delta ADP) = \frac{1}{2} \cdot AD \cdot AP = \frac{1}{2} \cdot x \cdot \sqrt{400 - 40x}$

b. $Opp' = \frac{1}{2} \cdot \sqrt{400 - 40x} + \frac{1}{2}x \cdot \dfrac{1}{2\sqrt{400 - 40x}} \cdot -40$

$= \frac{1}{2}\sqrt{400 - 40x} - \dfrac{10x}{\sqrt{400 - 40x}} = 0$

$$\tfrac{1}{2}\sqrt{400-40x} = \frac{10x}{\sqrt{400-40x}}$$

$$\tfrac{1}{2}(400-40x) = 10x$$

$$200-20x = 10x$$

$$-30x = -200$$

$$x = 6\tfrac{2}{3}$$

$$y = 38{,}49 \text{ cm}^2$$

15.3 Trillingen

Opgave 29:

a. $per = 5$

$c = \frac{2\pi}{per} = \frac{2\pi}{5} = \frac{2}{5}\pi$

$amp = 3$

b. $y_{P'} = 3\sin(\frac{2}{5}\pi\, t)$

Opgave 30:

a. $per = \frac{2\pi}{c} = \frac{2\pi}{40\pi} = 0{,}05$

$freq = \frac{1}{per} = \frac{1}{0{,}05} = 20\text{ Hz}$

b. $u_Q = 5\sin(40\pi\, t - \frac{3}{5}\pi) = 5\sin(40\pi(t - 0{,}015))$

$\frac{0{,}015}{0{,}05} = 0{,}3$ dus de faseachterstand is $\frac{3}{10}$

c. translatie $(0.015,0)$

d. per trilling 4x amplitude $4 \cdot 5 = 20\text{ cm}$
1 kwartier = 900 sec
dus per kwartier $900 \cdot 20 = 18000$ trillingen
$\text{afstand} = 18000 \cdot 20 = 360000\text{ cm} = 3{,}6\text{ km}$

Opgave 31:

a. $per = \frac{1}{freq} = \frac{1}{3}$

$c = \frac{2\pi}{per} = \frac{2\pi}{1/3} = 6\pi$

$u_P = 10\sin(6\pi\, t)$

faseachterstand van $\frac{1}{10}$ dus $\frac{1}{10} \cdot per = \frac{1}{10} \cdot \frac{1}{3} = \frac{1}{30}$

$u_Q = 10\sin(6\pi(t - \frac{1}{30}))$

b. $10\sin(6\pi\, t) = 10\sin(6\pi(t - \frac{1}{30}))$

$6\pi\, t = 6\pi(t - \frac{1}{30}) + k \cdot 2\pi \quad \vee \quad 6\pi\, t = \pi - 6\pi(t - \frac{1}{30}) + k \cdot 2\pi$

geen oplossingen $\qquad 6\pi\, t = \pi - 6\pi\, t + \frac{1}{5}\pi + k \cdot 2\pi$

$12\pi\, t = 1\frac{1}{5}\pi + k \cdot 2\pi$

$t = \frac{1}{10} + k \cdot \frac{1}{6}$

$t = \frac{1}{10} \quad \vee \quad t = \frac{4}{15} \quad \vee \quad t = \frac{13}{30} \quad \vee \quad t = \frac{3}{5} \quad \vee \quad t = \frac{23}{30} \quad \vee \quad t = \frac{14}{15}$

c. $t = \frac{4}{15} \quad \vee \quad t = \frac{3}{5} \quad \vee \quad t = \frac{14}{15}$

Opgave 32:

a. $x_{P''} = x_P = b\cos(ct) = b\sin(ct + \frac{1}{2}\pi) = b\sin(c(t + \frac{1}{2c}\pi))$

b. als je de figuur $45°$ draait, zie je dat de projectie van P op de lijn $y = x$ op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y-as.
Dus de project van P op de lijn $y = x$ voert een harmonische trilling uit.

Opgave 33:

a. boog $BC = \frac{10}{360} \cdot 2\pi \cdot 1 = 0{,}1745\,\text{m}$

b. $\sin 5° = \frac{A'B}{1}$

$A'B = \sin 5° = 0{,}0716$

$BC = 2 \cdot A'B = 0{,}1743\,\text{m}$

c. $b = 0{,}07$

$T = 2\pi\sqrt{\frac{1}{9{,}81}} = 2{,}01$

$c = \frac{2\pi}{T} = \frac{2\pi}{2{,}01} = 3{,}13$

d. $T = 2$ dus per periode 2x door de evenwichts-as, dus per seconde één tik

Opgave 34:

$$\left.\begin{array}{l} \max = 6{,}766 \\ \min = -6{,}766 \end{array}\right\} \quad amp = 6{,}77$$

$d = 0{,}15$

$u = 6{,}77\sin(2(t - 0{,}15))$

Opgave 35:

a. $\begin{cases} t + u = a \\ t - u = b \quad + \end{cases}$

$2t = a + b$

$t = \frac{1}{2}(a + b)$

$u = a - t = a - \frac{1}{2}(a + b) = a - \frac{1}{2}a - \frac{1}{2}b = \frac{1}{2}(a - b)$

b.
$$\begin{aligned} \sin(a) + \sin(b) &= \sin(t + u) + \sin(t - u) \\ &= \sin t \cdot \cos u + \cos t \cdot \sin u + \sin t \cdot \cos u - \cos t \cdot \sin u \\ &= 2\sin t \cdot \cos u \\ &= 2\sin(\tfrac{1}{2}(a + b)) \cdot \cos(\tfrac{1}{2}(a - b)) \end{aligned}$$

Opgave 36:

$$\begin{aligned} \sin a - \sin b &= \sin(t + u) - \sin(t - u) \\ &= \sin t \cdot \cos u + \cos t \cdot \sin u - (\sin t \cdot \cos u - \cos t \cdot \sin u) \\ &= 2\cos t \cdot \sin u \\ &= 2\cos(\tfrac{1}{2}(a + b)) \cdot \sin(\tfrac{1}{2}(a - b)) \end{aligned}$$

$$\begin{aligned} \cos a + \cos b &= \cos(t + u) + \cos(t - u) \\ &= \cos t \cdot \cos u - \sin t \cdot \sin u + \cos t \cdot \cos u + \sin t \cdot \sin u \\ &= 2\cos t \cdot \cos u \\ &= 2\cos(\tfrac{1}{2}(a + b)) \cdot \cos(\tfrac{1}{2}(a - b)) \end{aligned}$$

$$\begin{aligned} \cos a - \cos b &= \cos(t + u) - \cos(t - u) \\ &= \cos t \cdot \cos u - \sin t \cdot \sin u - (\cos t \cdot \cos u + \sin t \cdot \sin u) \\ &= -2\sin t \cdot \sin u \\ &= -2\sin(\tfrac{1}{2}(a + b)) \cdot \sin(\tfrac{1}{2}(a - b)) \end{aligned}$$

Opgave 37:

a. $\max = 5{,}69 \quad \min = -5{,}69$ dus $b = 5{,}69$

$c = 500\pi$

beginpunt: $t = 4{,}66 \cdot 10^{-4}$ dus $d = 500\pi \cdot 4{,}66 \cdot 10^{-4} = 0{,}73$

$u = 5{,}69\sin(500\pi t - 0{,}73)$

b. $per = \frac{2\pi}{500\pi} = 0{,}004$

$freq = \frac{1}{per} = \frac{1}{0{,}004} = 250\,\text{Hz}$

per trilling 4x amplitude $= 4 \cdot 5{,}69 = 22{,}76$

per seconde $250 \cdot 22{,}76 = 5690\,\text{mm} = 5{,}69\,\text{m}$

c. $u' = 500\pi \cdot 5{,}69\cos(500\pi t - 0{,}73)$

$u'(0) = 6660\,{}^{mm}\!/_{\text{sec}} = 6{,}66\,{}^{m}\!/_{s} = 24\,{}^{km}\!/_{uur}$

Opgave 38:

a. $u = 3\sin(500\pi t) + 3\sin(500\pi t - \frac{1}{2}\pi)$

$= 3(\sin(500\pi t) + \sin(500\pi t - \frac{1}{2}\pi))$

$= 3 \cdot 2 \cdot \sin(500\pi t - \frac{1}{4}\pi) \cdot \cos(\frac{1}{4}\pi)$

$3\sqrt{2} \cdot \sin(500\pi t - \frac{1}{4}\pi)$

b. de maximale snelheid is in het snijpunt met de evenwichts-as

$500\pi t - \frac{1}{4}\pi = 0$

$500\pi t = \frac{1}{4}\pi$

$t = \frac{1}{2000}$

$u' = 500\pi \cdot 3\sqrt{2} \cdot \cos(500\pi t - \frac{1}{4}\pi)$

$u'(\frac{1}{2000}) = 1500\pi\sqrt{2}\,{}^{mm}\!/_{\text{sec}} = 6664\,{}^{mm}\!/_{\text{sec}} = 6{,}66\,{}^{m}\!/_{\text{sec}} = 24\,{}^{km}\!/_{uur}$

Opgave 39:

$u = p\sin(ct) + p\sin(ct - d)$

$= p(\sin(ct) + \sin(ct - d))$

$= p \cdot 2\sin(ct - \frac{1}{2}d)\cos(\frac{1}{2}d)$

$= 2p\cos(\frac{1}{2}d)\sin(ct - \frac{1}{2}d)$

$= b\sin(ct - \frac{1}{2}d)$

Opgave 40:

a. $u_1 = \sin t + \cos t$

$= \sin t + \sin(t + \frac{1}{2}\pi)$

$= 2\sin(t + \frac{1}{4}\pi)\cos(-\frac{1}{4}\pi)$

$= \sqrt{2} \cdot \sin(t + \frac{1}{4}\pi)$

b. $\max = 2{,}24$ en $\min = -2{,}24$ dus $b = 2{,}24$

beginpunt: $d = 5{,}18$

$u_2 = 2{,}24\sin(t - 5{,}18)$

Opgave 41:

a. de periode van $\sin(2t)$ is $\frac{2\pi}{2}=\pi$ en de periode van $\sin(3t)$ is $\frac{2\pi}{3}=\frac{2}{3}\pi$
het kleinste getal waar beiden een geheel aantal keer in passen is 2π .

b. de periode van $\sin(2t)$ is $\frac{2\pi}{2}=\pi$ en de periode van $\sin(4t)$ is $\frac{2\pi}{4}=\frac{1}{2}\pi$
het kleinste getal waar beiden een geheel aantal keer in passen is π .

Opgave 42:

a.	$\sin(100\pi t)$	$\sin(101\pi t)$
op $[0,2\pi]$ heeft	100π golven	101π golven
op $[0,2]$ heeft	100 golven	101 golven
dus de periode is 2		
b.	$\sin(100t)$	$\sin(101t)$
op $[0,2\pi]$ heeft	100 golven	101 golven
dus de periode is 2π		
c.	$\sin(100\pi t)$	$\sin(105\pi t)$
op $[0,2\pi]$ heeft	100π golven	105π golven
op $[0,2]$ heeft	100 golven	105 golven
op $[0,\frac{2}{5}]$ heeft	20 golven	21 golven
dus de periode is $\frac{2}{5}$		
d.	$\sin(\frac{1}{4}\pi t)$	$\sin(\frac{1}{5}\pi t)$
op $[0,2\pi]$ heeft	$\frac{1}{4}\pi$ golven	$\frac{1}{5}\pi$ golven
op $[0,2]$ heeft	$\frac{1}{4}$ golf	$\frac{1}{5}$ golf
op $[0,40]$ heeft	5 golven	4 golven
dus de periode is 40		

Opgave 43:

	$\sin(660\pi t)$	$\sin(661\pi t)$
op $[0,2\pi]$ heeft	660π golven	661π golven
op $[0,2]$ heeft	660 golven	661 golven
dus de periode is 2		

Opgave 44:

a. $0{,}2\sin(1400\pi t)$: $per=\frac{2\pi}{1400\pi}=\frac{1}{700}$

$$freq=\frac{1}{\frac{1}{700}}=700\text{ Hz}$$

$0{,}3\sin(2100\pi t)$: $per=\frac{2\pi}{2100\pi}=\frac{1}{1050}$

$$freq=\frac{1}{\frac{1}{1050}}=1050\text{ Hz}$$

$0{,}1\sin(2800\pi t)$: $per=\frac{2\pi}{2800\pi}=\frac{1}{1400}$

$$freq=\frac{1}{\frac{1}{1400}}=1400\text{ Hz}$$

b.

	$\sin(700\pi t)$	$\sin(1400\pi t)$	$\sin(2100\pi t)$	$\sin(2800\pi t)$
op $[0,2\pi]$	700π	1400π	2100π	2800π
op $[0,2]$	700	1400	2100	2800
op $[0,\frac{1}{50}]$	7	14	21	28
op $[0,\frac{1}{350}]$	1	2	3	4

dus de periode is $\frac{1}{350}$

Opgave 45

a.

	$\sin(500\pi t)$	$\sin(550\pi t)$
op $[0,2\pi]$	500π	550π
op $[0,2]$	500	550
op $[0,0.04]$	10	11

dus de periode is 0,04

b. $X \max = 0{,}06$

c. $u_5 = 0{,}6\sin(500\pi t) + 0{,}6\sin(500\pi t - 0{,}5\pi)$
$= 0{,}6(\sin(500\pi t) + \sin(500\pi t - 0{,}5\pi))$
$= 0{,}6 \cdot 2\sin(500\pi t - 0{,}25\pi)cis(0{,}25\pi)$
$= 0{,}6\sqrt{2} \cdot \sin(500\pi t - 0{,}25\pi)$

15.4 Lissajous-figuren.

Opgave 46:

a. $x(\frac{1}{4}\pi) = \sin\frac{1}{4}\pi = \frac{1}{2}\sqrt{2}$

$y(\frac{1}{4}\pi) = \sin\frac{1}{2}\pi = 1$

dus voor $t = \frac{1}{4}\pi$ krijg je het punt $(\frac{1}{2}\sqrt{2},1)$

b. $t = 0 \quad x = \sin 0 = 0$

$y = \sin 0 = 0$

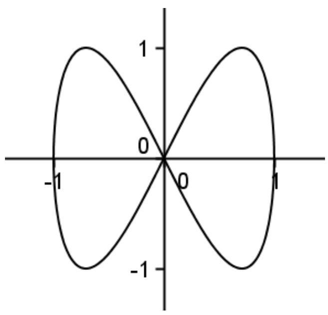
$t = \frac{1}{2}\pi \quad x = \sin\frac{1}{2}\pi = 1$

$y = \sin\pi = 0$

c.

t	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$1\frac{1}{4}\pi$	$1\frac{1}{2}\pi$	$1\frac{3}{4}\pi$	2π
x	0	$\frac{1}{2}\sqrt{2}$	1	$\frac{1}{2}\sqrt{2}$	0	$-\frac{1}{2}\sqrt{2}$	-1	$-\frac{1}{2}\sqrt{2}$	0
y	0	1	0	-1	0	1	0	-1	0

d.



Opgave 47:

a. er is 1 punt met $x = 1$ en 1 punt met $x = -1$ en de grafiek snijdt de evenwichts-as 2 keer
er zijn 4 punten met $y = 1$ en 4 punten met $y = -1$ en de grafiek snijdt de evenwichts-as 8 keer

b. $a = 2 \quad b = 8$

c. $a = 3 \quad b = 12$

Opgave 48:

$c = 2$

Opgave 49:

$a = 2 \quad b = 5$

Opgave 50:

a. $a = 2 \quad b = 3$

b. snijpunt x-as: $t = 0 \quad \vee \quad t = \frac{2}{5}\pi \quad \vee \quad t = \frac{4}{5}\pi \quad \vee \quad t = 1\frac{1}{5}\pi \quad \vee \quad t = 1\frac{3}{5}\pi \quad \vee \quad t = 2\pi$

$x = 1: \quad t = \frac{1}{4}\pi \quad \vee \quad t = 1\frac{1}{4}\pi$

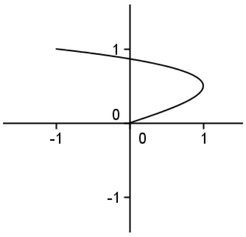
$x = -1: \quad t = \frac{3}{4}\pi \quad \vee \quad t = 1\frac{3}{4}\pi$

$y = 1: \quad t = \frac{1}{6}\pi \quad \vee \quad t = \frac{5}{6}\pi \quad \vee \quad t = 1\frac{1}{2}\pi$

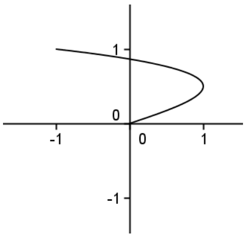
$y = -1: \quad t = \frac{1}{2}\pi \quad \vee \quad t = 1\frac{1}{6}\pi \quad \vee \quad t = 1\frac{5}{6}\pi$

Opgave 51:

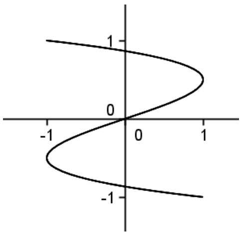
a.



b.



c.



2 keer doorlopen

d. $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$

Opgave 52:

a. $c = 5$

b. op $[\frac{1}{2}\pi, 1\frac{1}{2}\pi]$ loopt de grafiek van rechts naar links
op $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ loopt de grafiek van links naar rechts

Opgave 53:

a. $y = 0$ dus $\sin 2t = 0$

$2t = 0 + k \cdot \pi$

$t = 0 + k \cdot \frac{1}{2}\pi$

$A(-\frac{1}{2},0) \quad D(\frac{1}{2}\sqrt{3},0) \quad G(\frac{1}{2},0) \quad I(-\frac{1}{2}\sqrt{3},0)$

$x = 0$ dus $\sin(t - \frac{1}{6}\pi) = 0$

$t - \frac{1}{6}\pi = 0 + k \cdot \pi$

$t = \frac{1}{6}\pi + k \cdot \pi$

$B(0, \frac{1}{2}\sqrt{3})$

$y = 1$ dus $\sin 2t = 1$

$2t = \frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{1}{4}\pi + k \cdot \pi$

$C(\sin \frac{1}{12}\pi, 1) \quad H(\sin 1\frac{1}{12}\pi, 1)$

$y = -1$ dus $\sin 2t = -1$

$2t = 1\frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{3}{4}\pi + k \cdot \pi$

$F(\sin \frac{7}{12}\pi, -1) \quad K(\sin 1\frac{7}{12}\pi, -1)$

$x=1$ dus $\sin(t-\frac{1}{6}\pi)=1$

$t-\frac{1}{6}\pi=\frac{1}{2}\pi+k\cdot 2\pi$

$t=\frac{2}{3}\pi+k\cdot 2\pi$

$E(1,-\frac{1}{2}\sqrt{3})$

$x=-1$ dus $\sin(t-\frac{1}{6}\pi)=-1$

$t-\frac{1}{6}\pi=1\frac{1}{2}\pi+k\cdot 2\pi$

$t=1\frac{2}{3}\pi+k\cdot 2\pi$

$J(-1,-\frac{1}{2}\sqrt{3})$

b. $\sin(t-\frac{1}{6}\pi)=\frac{1}{2}$

$t-\frac{1}{6}\pi=\frac{1}{6}\pi \quad \vee \quad t-\frac{1}{6}\pi=\frac{5}{6}\pi$

$t=\frac{1}{3}\pi \quad \vee \quad t=\pi$

$y=\frac{1}{2}\sqrt{3} \qquad y=0$

$L=\frac{1}{2}\sqrt{3}$

c. $\sin 2t=\sin(t-\frac{1}{6}\pi)$

$2t=t-\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad 2t=\pi-(t-\frac{1}{6}\pi)+k\cdot 2\pi$

$t=-\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad 2t=\pi-t+\frac{1}{6}\pi+k\cdot 2\pi$

$t=-\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad 3t=\frac{7}{6}\pi+k\cdot 2\pi$

$t=-\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad t=\frac{7}{18}\pi+k\cdot\frac{2}{3}\pi$

$t=1\frac{5}{6}\pi \quad \vee \quad t=\frac{7}{18}\pi \quad \vee \quad t=\frac{19}{18}\pi \quad \vee \quad t=\frac{31}{18}\pi$

Opgave 54:

$a=3$

$y_B=\sin(\frac{3}{4}\pi+b)=0$

$\frac{3}{4}\pi+b=0 \quad \vee \quad \frac{3}{4}\pi+b=\pi$

$b=-\frac{3}{4}\pi \quad \vee \quad b=\frac{1}{4}\pi$

$b=-\frac{3}{4}\pi$ levert de gespiegelde grafiek op dus alleen $b=\frac{1}{4}\pi+k\cdot 2\pi$

Opgave 55:

a. $x=\sin 2t=0$

$2t=0+k\cdot\pi$

$t=0+k\cdot\frac{1}{2}\pi$

$A(0,\frac{1}{2}\sqrt{3}) \quad B(0,\frac{1}{2}) \quad C(0,-\frac{1}{2}) \quad D(0,-\frac{1}{2}\sqrt{3})$

b. $x=\sin 2t=-\frac{1}{2}$

$2t=1\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad 2t=1\frac{5}{6}\pi+k\cdot 2\pi$

$t=\frac{7}{12}\pi+k\cdot\pi \quad \vee \quad t=\frac{11}{12}\pi+k\cdot\pi$

$t=\frac{11}{12}\pi \qquad t=1\frac{11}{12}\pi$

$y_A=\sin 1\frac{1}{4}\pi=-\frac{1}{2}\sqrt{2} \qquad y_E=\sin 2\frac{1}{4}\pi=\frac{1}{2}\sqrt{2}$

$EH=\sqrt{2}$

Opgave 56:

a. $\begin{cases} x = r \cdot \cos t = r \cdot \sin(t + \frac{1}{2}\pi) \\ y = r \cdot \sin t \end{cases}$

b. $\begin{cases} x = \sin(t + \frac{1}{2}\pi) \\ y = r \cdot \sin t \end{cases}$

Opgave 57:

Door $(0,-1)$ dus $-1 = 0 + q$
dus $q = -1$
Door $(1,1)$ dus $1 = p - 1$
dus $p = 2$

Opgave 58:

$x = \sin(t - \frac{1}{4}\pi) = \sin t \cdot \cos \frac{1}{4}\pi - \cos t \cdot \sin \frac{1}{4}\pi$
$= \frac{1}{2}\sqrt{2} \cdot \sin t - \frac{1}{2}\sqrt{2} \cdot \cos t$
$= \frac{1}{2}\sqrt{2}(\sin t - \cos t)$

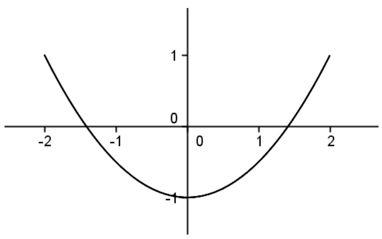
$x^2 = \frac{1}{2}(\sin t - \cos t)^2$

$-2x^2 = -(\sin t - \cos t)^2$
$= -\sin^2 t + 2\sin t \cos t - \cos^2 t$
$= -1 + 2\sin t \cos t$

$-2x^2 + 1 = -1 + 2\sin t \cos t + 1$
$= 2\sin t \cos t$
$= \sin 2t = y$

Opgave 59:

a.

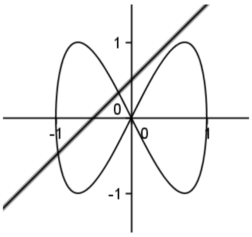


de keerpunten zijn $(-2,1)$ en $(2,1)$

b. $y = ax^2 + b$ door $(0,-1)$
dus $b = -1$
$y = ax^2 - 1$ door $(2,1)$
$1 = 4a - 1$
$-4a = -2$
$a = \frac{1}{2}$
$K: \quad y = \frac{1}{2}x^2 - 1$
$\frac{1}{2}x^2 - 1 = \frac{1}{2}(2\sin t)^2 - 1 = 2\sin^2 t - 1$
$y = \sin(2t - \frac{1}{2}\pi) = \sin 2t \cdot \cos \frac{1}{2}\pi - \cos 2t \cdot \sin \frac{1}{2}\pi = 0 - \cos 2t = -(1 - 2\sin^2 t) = 2\sin^2 t - 1$
dus $y = 2\sin^2 t - 1 = \frac{1}{2}x^2 - 1$

Opgave 60:

a.



b. $\sin 2t = \sin t + \frac{1}{2}$

$y_1 = \sin 2x \quad y_2 = \sin x + \frac{1}{2}$ intersect geeft $x = 3{,}31 \;\vee\; x = 4{,}96$

dus $t = 3{,}31 \;\vee\; t = 4{,}96$

$t = 3{,}31$ geeft $(-0{,}17; 0{,}33)$

$t = 4{,}96$ geeft $(-0{,}97; -0{,}47)$

c. $4x^2 - 4x^4 = 4\sin^2 t - 4\sin^4 t$

$= 4\sin^2 t \cdot (1 - \sin^2 t)$

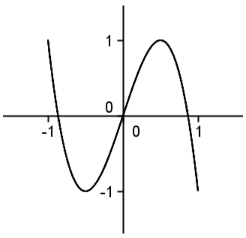
$= 4\sin^2 t \cdot \cos^2 t$

$y^2 = (\sin 2t)^2 = (2\sin t \cos t)^2 = 4\sin^2 t \cdot \cos^2 t$

$y^2 = 4\sin^2 t \cdot \cos^2 t = 4x^2 - 4x^4$

Opgave 61:

a.



de keerpunten zijn $(-1,1)$ en $(1,-1)$

b. $3x - 4x^3 = 3\sin t - 4\sin^3 t$

$y = \sin 3t = \sin(2t + t)$

$= \sin 2t \cdot \cos t + \cos 2t \cdot \sin t$

$= 2\sin t \cos t \cdot \cos t + (1 - 2\sin^2 t) \cdot \sin t$

$= 2\sin t \cdot \cos^2 t + \sin t - 2\sin^3 t$

$= 2\sin t \cdot (1 - \sin^2 t) + \sin t - 2\sin^3 t$

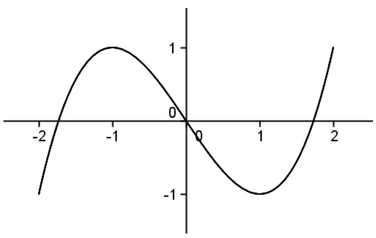
$= 2\sin t - 2\sin^3 t + \sin t - 2\sin^3 t$

$= 3\sin t - 4\sin^3 t$

$= 3x - 4x^3$

Opgave 62:

a.



b. $y = ax^3 + bx$ door $(2,1)$ en $(1,-1)$ dus

$$\begin{cases} 1 = 8a + 2b & \times 1 \\ -1 = a + b & \times 2 \end{cases}$$

$$\begin{cases} 1 = 8a + 2b \\ -2 = 2a + 2b & - \end{cases}$$

$3 = 6a$

$a = \frac{1}{2}$

$b = -1\frac{1}{2}$

$c = -2$ en $d = 2$

$K: \quad y = \frac{1}{2}x^3 - 1\frac{1}{2}x$

$\frac{1}{2}x^3 - 1\frac{1}{2}x = \frac{1}{2}(2\cos t)^3 - 1\frac{1}{2} \cdot 2\cos t = 4\cos^3 t - 3\cos t$

$$\begin{aligned} y = \cos 3t &= \cos(2t + t) \\ &= \cos 2t \cdot \cos t - \sin 2t \cdot \sin t \\ &= (2\cos^2 t - 1) \cdot \cos t - 2\sin t \cos t \cdot \sin t \\ &= 2\cos^3 t - \cos t - 2\sin^2 t \cdot \cos t \\ &= 2\cos^3 t - \cos t - 2 \cdot (1 - \cos^2 t) \cdot \cos t \\ &= 2\cos^3 t - \cos t - 2\cos t + 2\cos^3 t \\ &= 4\cos^3 t - 3\cos t \\ &= \tfrac{1}{2}x^3 - 1\tfrac{1}{2}x \end{aligned}$$